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Low-frequency second-order wave-drift forces and damping

AAD J. HERMANS
Faculty of Information Technology and Systems, TUDelft, Mekelweg 4, 2628 CD Delft, The Netherlands.
e-mail: A.J.Hermans@math.tudelft.nl

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Abstract. The effects of free-surface waves on floating structures are of great importance in the offshore industry. Besides the first-order responses to the waves, second-order effects play a role, in particular in the situation where an anchored object may be excited in its frequency of resonance. This paper concerns the study of this phenomenon. Two different approaches are developed, each with its own advantage. Special attention is paid to excitation forces generating the motions; also a theory to determine the inviscid damping that limits the extreme excursions is discussed. Numerical results are presented for a sphere, and numerical results for two classes of tankers, namely for a VLCC and a LNG-carrier, and a semi-submersible are compared with experimental data obtained at the Maritime Research Institute in the Netherlands (MARIN).

Keywords: hydrodynamics, wave-drift forces, wave-drift damping, integral equations, panel method.

1. Introduction

Traditionally there is interest in the second-order forces acting on ships sailing with a constant forward speed in a wave field. The resulting added resistance is of importance because it generates a substantial speed loss of the vessel. In 1924 Suyehiro [1] discussed the drift of ships caused by rolling among waves. He measured the steady drift force experienced by a ship while rolling in waves. Later on Watanabe [2] and Havelock [3, 4, 5] studied these phenomena and laid the basis for a mathematical theory. Finally, in the paper of Maruo [6] in 1960 a thorough mathematical theory is formulated. Newman [7] extended the theory to slender ships and compared with experimental results. Faltinsen and Michelsen [8] extended the formulation to the case of finite water depth at zero forward speed.

About thirty years ago the tanker size increased and it became necessary to load and unload in open sea near the coast. It was noticed that the resonance of the mooring could hamper the operation severely. Although the resonance frequency of the moored system was well below the frequency band of the incident wave field, severe excitations could occur. In 1970 Hsu and Blenkarn [9] studied this phenomenon and Remery and Hermans [10] performed some additional tests to shed light on the source of this phenomenon. They also studied the underlying physical process and presented a preliminary theory on slow drift oscillations. The first suggestion was to use the constant drift force obtained by Newman [7] and Faltinsen and Michelsen [8] for the excitation force at low frequency. This leads to satisfying results in many cases. However, to cover all situations the low-frequency drift force has to be computed. In 1980 Pinkster [11] completed the theory and computation of the low-frequency second-order wave-exciting *forces* on floating structures. The second-order effects due to the first-order potential and motions were taken completely into account in his computations, but the effect of the second-order potential consisted of a rough approximation. Meanwhile many authors studied this phenomenon.

A problem addressed and solved provisionally in the paper of Remery and Hermans [10] concerns the damping at the resonance frequency. An extensive study of this phenomenon was published in 1988 by Wichers [12]. The main result is that for moderate sea states the damping is dominated by viscous effects, however, in survival states the wave-drift damping due to the velocity dependency of the wave-drift force becomes dominant. A suggestion to compute this damping by means of a perturbation approach was presented in 1985 by Huijsmans and Hermans [13]. Hermans [14] and Grue and Palm [15] applied the approach of Newman [7] to compute the speed-dependent wave-drift force by which the wave-drift damping can be computed. In the derivation of Hermans [14] a slight error occurs, but finally the two different derivations lead to the same analytic expression for the second-order wave force influenced by a small constant velocity field [16]. Especially in the frequency range where the first-order motions are dominant, the second-order forces must be calculated taking into account the complete first-order excitation and reaction potentials at low speed; also, for finite water depth the second-order potential may lead to a contribution that cannot be ignored. Grue [17] shows that for the computation of second order moments the second-order potential cannot be ignored.

2. Mathematical formulation of the potentials

In the literature a variety of methods to compute the first-order potentials can be found. For zero forward speed the most commonly used method is based on the application of Green's theorem to the fluid domain, or based on a source distribution, using the harmonic Green's function, which obeys the linearised free-surface condition. Some computer codes are commercially available; they treat both the infinite-depth or the finite-depth case. Newman [18, 19] and Noblesse [20], independently, made ingenious fast codes to compute the Green's function and its derivatives. The sources of the codes are either commercially or freely available. The panel-method codes developed with these source functions vary widely from zero order (constant) to higher-order methods; there are also codes making use of spline approximations.

Meanwhile codes are developed that make use of simple Rankine, $1/r$ sources, so they consist of source distributions over the object and the free surface; one of the first successful efforts can be found in the work of Yeung [21]. This can be done in the frequency-domain or in the time-domain. The linearised versions consider the free-surface source distribution along $z = 0$, while the nonlinear versions may have a distribution at the actual free surface. In the latter case the free surface is determined iteratively. Also, the source strength on a panel can be described by lower- or higher-order appoximations.

To compute the diffraction of waves in a current or by a steadily moving object there is a Green's function available that obeys the linearised free-surface condition for harmonic waves in a uniformly constant flow. Some attemps to use this Green's function in a similar code as described above did not give rise to a substantional improvement of the results obtained by the classical widely available strip-theory. This strip-theory, in principle, makes use of the slenderness or thinness. Hence, the stationary potential is approximated by the unperturbed flow. This theory leads to an approach that is two-dimensional, sideways, in nature. Three-dimensional effects are taken into account by means of the interaction of adjacent strips. The modified strip theory gives fairly accurate results for a large class of ships. However, in the

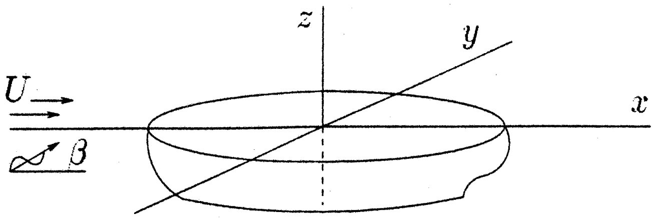

Figure 1. Coordinate system.

case of full-bodied ships and offshore structures this appraoch is not reliable. The frequency-domain and time-domain methods based on a superposition of the double body potential and the wave potential may lead to a significant improvement.

In the next subsections we discuss two different ways to compute the first-order diffraction and radiation potential superimposed on the double-body potential. These approaches are suitable for low to moderate forward velocity or current. At high speed it is expected that these methods have to improve. We expect that for extreme cases, where the steady wave pattern is significant, even for low wave heights, the wave potential has to be superimposed on top of the nonlinear steady potential. Some preliminary results are available where the RAPID results of Raven [22] are used as steady potential.

2.1. Slow-speed approximation

We first derive the equations for the potential function $\Phi(\boldsymbol{x}, t)$, such that the fluid velocity $\boldsymbol{u}(\boldsymbol{x}, t)$ is defined as $\boldsymbol{u}(\boldsymbol{x}, t) = \nabla\Phi(\boldsymbol{x}, t)$. The total potential function will be split up in a steady and a non-steady part in a well-known way

$$\Phi(\boldsymbol{x}, t) = Ux + \overline{\phi}(\boldsymbol{x}; U) + \tilde{\phi}(\boldsymbol{x}, t; U).$$

In this formulation U is the incoming unperturbed velocity field, obtained by considering a coordinate system fixed to the ship moving under a drift angle α. In our approach this angle need not be small. The time-dependent part of the potential consists of an incoming, diffracted and radiated (for six modes of motion) wave

$$\tilde{\phi}(\boldsymbol{x}, t; U) = \tilde{\phi}^{\text{inc}}(\boldsymbol{x}, t; U) + \tilde{\phi}^{(7)}(\boldsymbol{x}, t; U) + \sum_{i=1}^{6} \tilde{\phi}^{(i)}(\boldsymbol{x}, t; U),$$

at frequency $\omega = \omega_0 + k_0 U \cos\beta$, where ω_0 and $k_0 = \omega_0^2/g$ are the frequency and wave number in the earth-fixed coordinate system, while ω is the frequency in the coordinate system fixed to the ship, see Figure 1. The waves are incoming under an angle β, with respect to the current. To compute the wave-drift forces all these components will be taken into account.

The equations for the total potential Φ can be written as: $\Delta\Phi = 0$ in the fluid domain D_e. At the free surface we have the dynamic and kinematic boundary conditions

$$\left.\begin{aligned} g\zeta + \Phi_t + \tfrac{1}{2}\nabla\Phi\cdot\nabla\Phi &= \mathcal{C}_{st} \\ \Phi_z - \Phi_x\zeta_x - \Phi_y\zeta_y - \zeta_t &= 0 \end{aligned}\right\} \quad \text{at } z = \zeta. \tag{1}$$

At the body surface we have

$$\frac{\partial \Phi}{\partial n} = \boldsymbol{V} \cdot \boldsymbol{n},$$

where $\boldsymbol{V}$ is the body velocity relative to the average body-fixed coordinate system.
We assume that the waves are high compared to the Kelvin stationary wave pattern, but that they are both small in nature, hence the free-surface boundary condition can be expanded at $z = 0$. We first eliminate ζ, which leads to the following boundary condition

$$\frac{\partial^2}{\partial t^2}\Phi + g\frac{\partial}{\partial z}\Phi + \frac{\partial}{\partial t}(\nabla\Phi \cdot \nabla\Phi) + \tfrac{1}{2}\nabla\Phi \cdot \nabla[\nabla\Phi \cdot \nabla\Phi] = 0 \quad \text{at } z = \zeta. \tag{2}$$

To compute the first-order wave potential the free surface has to be linearised first. We assume $\tilde{\phi}(\boldsymbol{x}, t; U) = \phi(\boldsymbol{x}; U)\exp(-i\omega t)$, then for $i = 1, \ldots, 6$ the free-surface condition at $z = 0$ can be written as

$$-\omega^2\phi - 2i\omega U\phi_x + U^2\phi_{xx} + g\phi_z = \mathcal{D}(U; \overline{\phi})\{\phi\} \quad \text{at } z = 0, \tag{3}$$

while for the diffracted potential $\phi^{(7)}$ the last term has to be replaced by $\mathcal{D}(U; \overline{\phi})\{\phi^{\text{inc}} + \phi^{(7)}\}$ and where $\mathcal{D}(U; \overline{\phi})$ is the following linear differential operator acting on ϕ. The quadratic terms in ϕ are neglected. Thus

$$\begin{aligned}\mathcal{D}(U; \overline{\phi})\{\phi\} \;=\; & -i\omega\{(\overline{\phi}_{xx} + \overline{\phi}_{yy})\phi + 2\nabla\overline{\phi} \cdot \nabla\phi\} \\ & +(2U\overline{\phi}_x + \overline{\phi}_x^2)\phi_{xx} + 2(U + \overline{\phi}_x)\overline{\phi}_y\phi_{xy} + \overline{\phi}_y^2\phi_{yy} \\ & +(3U\overline{\phi}_{xx} + \overline{\phi}_x\overline{\phi}_{xx} + \overline{\phi}_y\overline{\phi}_{xy})\phi_x + (2U\overline{\phi}_{xy} + \overline{\phi}_x\overline{\phi}_{xy} + \overline{\phi}_y\overline{\phi}_{yy})\phi_y.\end{aligned}$$

The linear problems for $\phi^{(i)}$ with $i = 1, \ldots, 7$ are solved by means of a source distribution along the ship hull, its waterline and the free surface $z = 0$. We write for each potential function

$$\begin{aligned}4\pi\phi(\boldsymbol{x}) \;=\; & -\iint_S \sigma(\boldsymbol{\xi})G(\boldsymbol{x}, \boldsymbol{\xi})\,\mathrm{d}S_\xi + \frac{U^2}{g}\int_{WL} \alpha_n\sigma(\boldsymbol{\xi})G(\boldsymbol{x}, \boldsymbol{\xi})\,\mathrm{d}s_\xi \\ & + \frac{i\omega}{g}\iint_{FS} G(\boldsymbol{x}, \boldsymbol{\xi})\mathcal{D}\{\phi\}\,\mathrm{d}S_\xi \quad \text{for } \boldsymbol{x} \in D_e.\end{aligned} \tag{4}$$

The function $G(\boldsymbol{x}, \boldsymbol{\xi})$ is the Green's function that obeys the free surface condition (3) with $\mathcal{D}$ equal to zero and $\alpha_n = \boldsymbol{e}_x \cdot \boldsymbol{n}$, where $\boldsymbol{e}_x$ equals the unit vector in the x-direction. In general, the boundary conditions on the ship are given in the form

$$\nabla\phi^{(i)} \cdot \boldsymbol{n} = V^{(i)}(\boldsymbol{x}) \quad \text{for } \boldsymbol{x} \in S \quad \text{and} \quad i = 1, \ldots, 7,$$

where $V^{(i)}$ is the normal velocity due to the motion in the ith mode with $i = 1, \ldots, 6$ and $V^{(7)}$ equals the normal velocity of the incident oscillating field. So $V^{(i)}$, with $i = 1, \ldots, 3$, correspond to the translation components $\boldsymbol{X} = \tilde{\boldsymbol{X}}\exp(-i\omega t)$ and, with $i = 4, \ldots, 6$, to components of the rotational motion $\boldsymbol{\Omega} = \tilde{\boldsymbol{\Omega}}\exp(-i\omega t)$ relative to the centre of gravity $\boldsymbol{x}_g$ of the body. The combined displacement vector is given by $\boldsymbol{\alpha} = \boldsymbol{X} + \boldsymbol{\Omega} \times (\boldsymbol{x} - \boldsymbol{x}_g) = \tilde{\boldsymbol{\alpha}} \exp(-i\omega t)$. In general notation we write

$$\frac{\partial \phi}{\partial n} = -i\omega\tilde{\boldsymbol{\alpha}} \cdot \boldsymbol{n} + [(\nabla(Ux + \overline{\phi}) \cdot \nabla)\tilde{\boldsymbol{\alpha}} - (\tilde{\boldsymbol{\alpha}} \cdot \nabla)\nabla(Ux + \overline{\phi})] \cdot \boldsymbol{n}.$$

This leads to an equation for the source strength, where we omitted the index i again

$$-2\pi\sigma(\boldsymbol{x}) - \iint_S \sigma(\boldsymbol{\xi})\frac{\partial}{\partial \boldsymbol{n}_x} G(\boldsymbol{x}, \boldsymbol{\xi})\, \mathrm{d}S_\xi + \frac{U^2}{g}\int_{WL} \alpha_n \sigma(\boldsymbol{\xi})\frac{\partial}{\partial \boldsymbol{n}_x} G(\boldsymbol{x}, \boldsymbol{\xi})\, \mathrm{d}s_\xi$$

$$+\frac{i\omega}{g}\iint_{FS} \frac{\partial}{\partial \boldsymbol{n}_x} G(\boldsymbol{x}, \boldsymbol{\xi})\mathcal{D}\{\phi\}\, \mathrm{d}S_\xi = 4\pi V(\boldsymbol{x}) \quad \text{for } \boldsymbol{x} \in S. \tag{5}$$

This equation can be solved iteratively in principle; however, an accurate numerical evaluation of the complete Green's function is rather elaborate. Therefore we make use of the fact that U is small, keeping in mind that there are two dimensionless parameters that play a role, namely $\tau = \omega U/g \ll 1$ and $\nu = gL/U^2 \gg 1$. The source potentials and the strengths can be evaluated as perturbation series with respect to τ

$$\sigma(\boldsymbol{\xi}) = \sigma_0(\boldsymbol{\xi}) + \tau\sigma_1(\boldsymbol{\xi}) + \hat{\sigma}(\boldsymbol{\xi}; U), \tag{6}$$

$$\phi(\boldsymbol{x}) = \phi_0(\boldsymbol{x}) + \tau\phi_1(\boldsymbol{x}) + \hat{\phi}(\boldsymbol{x}; U), \tag{7}$$

where $\hat{\sigma}$ and $\hat{\phi}$ are $O(\tau^2)$ as $\tau \to 0$, while the expansion of G is less trivial. We write

$$G(\boldsymbol{x}, \boldsymbol{\xi}; U) = -\frac{1}{\boldsymbol{r}} + \frac{1}{\boldsymbol{r}'} - \{\psi_0(\boldsymbol{x}, \boldsymbol{\xi}) + \tau\psi_1(\boldsymbol{x}, \boldsymbol{\xi}) + \cdots$$

$$\cdots \tilde{\psi}_0(\boldsymbol{x}, \boldsymbol{\xi}) + \nu^{-1}\tilde{\psi}_1(\boldsymbol{x}, \boldsymbol{\xi}) + \cdots\}. \tag{8}$$

The first term between brackets corresponds to the Green's function at zero forward speed, for which there exist several fast computer codes. The second term is the modification due to small values of the forward velocity. Computations can be carried out by means of a modification of the existing fast code. Nonuniformities can be taken care of as described by Huijsmans [23, 24]. The third term between brackets is the one that describes the Kelvin effect on the wave Green's function. In (Hermans [25]) we explained how one takes care of this term that is linear in ν and therefore tends to infinity as U goes to zero. In practice the first two terms are computed in the expansions of the potentials and the source strengths for the excitation and the six modes of the motion. This approach is easily applied to the situation of deep water. For finite water depth the evaluation of the Green's function for finite velocities leads to terms that are not as easy to compute as in the deep-water case, where all the expressions needed can be expressed in derivatives of the zero-speed Green's function.

2.2. Time-domain simulation

Recently Prins and Hermans [26, 27, 28, 29] developed a time-domain method to compute the first-order time-dependent part of the potential function. With this approach the complete potential function is written as follows

$$\Phi(\boldsymbol{x}, t) = \overline{\Phi}(\boldsymbol{x}) + \psi(\boldsymbol{x}, t).$$

For small values of the forward velocity the stationary potential $\overline{\Phi}(\boldsymbol{x})$ is approximated by the double-body potential with zero vertical velocity at the mean free surface $z = 0$. We insert this expression in Equation (2) and linearise the resulting expression at $z = 0$ and obtain

$$\frac{\partial^2 \psi}{\partial t^2} + g\frac{\partial \psi}{\partial z} + 2\nabla\overline{\Phi} \cdot \nabla\frac{\partial \psi}{\partial t} + \tfrac{1}{2}\nabla(\nabla\overline{\Phi} \cdot \nabla\overline{\Phi}) \cdot \nabla\psi + \nabla\overline{\Phi} \cdot \nabla(\nabla\overline{\Phi} \cdot \nabla\psi)$$
$$-\tfrac{1}{2}(\nabla\overline{\Phi} \cdot \nabla\overline{\Phi} - U^2)\left(\frac{\partial^2 \psi}{\partial z^2} + \frac{1}{g}\frac{\partial^3 \psi}{\partial t^2 \partial z}\right)$$
$$-\frac{\partial^2 \overline{\Phi}}{\partial z^2}\left(\nabla\overline{\Phi} \cdot \nabla\psi + \frac{\partial \psi}{\partial t}\right) = 0 \quad \text{at } z = 0. \tag{9}$$

The solution method is based on the applications of Green's theorem for the whole fluid domain

$$\tfrac{1}{2}\psi(\boldsymbol{x}, t) = \int_{\partial V}\left(\psi(\boldsymbol{\xi}, t)\frac{\partial G(\boldsymbol{x}, \boldsymbol{\xi})}{\partial n_\xi} - \frac{\partial \psi(\boldsymbol{\xi}, t)}{\partial n_\xi}G(\boldsymbol{x}, \boldsymbol{\xi})\right) \mathrm{d}S. \tag{10}$$

In this formulation the Green's function is chosen as the point-source at $\boldsymbol{x} = \boldsymbol{\xi}$ obeying the bottom condition, hence

$$G(\boldsymbol{x}, \boldsymbol{\xi}) = \frac{1}{4\pi}\left(\frac{1}{|\boldsymbol{x} - \boldsymbol{\xi}|} + \frac{1}{|\boldsymbol{x} - \boldsymbol{\xi}^*|}\right),$$

where $|\boldsymbol{x} - \boldsymbol{\xi}^*|$ is the distance to the source point reflected with respect to the flat bottom at $z = -h$. The integration is taken over the object, the free surface $z = 0$ and the outer closing boundaries. If we assume the time-dependent potential to be a superposition of the nondisturbed incident wave, the diffracted wave potential and the radiation (motion) potential we can compute the diffracted or the radiated by imposing the free surface condition and the body boundary condition. The closing boundaries need some extra care; we should impose a proper non-reflecting boundary condition. The differentiations along the free surface are handled by means of a central or upwind difference scheme, while the time differentiation is treated with an implicit difference scheme. In principle, we want to be able to compute the disturbance due to a general non-harmonic incident wave. We also want to keep the computing area as small as possible. This makes the choice of the non-reflecting boundary condition at the outer boundaries non-trivial. For harmonic waves and the boundaries far away we may choose a Sommerfeld-type condition, taking care of the two families of waves in the uniform current time harmonic case. This has been done by Prins [28]. Sierevogel and Hermans [30, 31] implemented a general formulation for the general case. Several methods are available, the method we implied is of the class of semi-discrete DtN-method, see Keller and Givoli [32]. The second-order time-derivative at the free surface is written by means of an explicit difference scheme at $t = (n + 1)\Delta t$ as follows

$$\psi_z^{n+1} + \mu\psi^{n+1} = \mathcal{L}(\psi^n, \psi^{n-1}, \ldots), \tag{11}$$

where $\mu = 2/g(\Delta t)^2$ and $\mathcal{L}(\psi^n, \psi^{n-1}, \ldots)$ is a linear operator depending on previous time steps only. Furthermore, the method is based on the application of Green's theorem to the exterior domain with a proper choice of the Green's function. The discretisation of the exterior free surface can be rather coarse, because the integration of the Green's function over an element is once done analytically. The potential is computed explicitely. This computation consists of a few matrix vector multiplications during each time step.

The number of unknowns in the equations is increased by a small amount, namely the number of elements at the outer boundaries. This approach is very efficient and it leads to a minor increase of computation time. The outer boundaries can be chosen at a rather small distance, depending on the wavelength and the steady velocity disturbance. An extensive study of non-reflecting boundary conditions for the acoustic case can be found in the book by Givoli [33].

3. Wave-drift forces and moments

In the zero-forward-speed case the constant second-order drift forces and moments can be computed as soon as the first-order potentials and motions are known. To compute the low-frequency drift forces and moments the second-order potential, depending on the difference frequency is also needed. Especially, if the water depth is finite, its contribution is considerable. If the forward speed is small, Grue [15] shows that the constant-drift moments depend on the second-order potential just as well. His conclusion that, depending on the way one calculates the constant drift forces, one has to include the second-order potential, is inconsistent. This is due to the treatment of free surface. In the derivation of the so-called far-field formulation one uses the correct no-flux condition at the exact free surface, while in the local pressure integration formulae are derived, making use of a free surface condition at $z = 0$. In the case of forward speed the second-order effects must be taken into account in the calculation of the low-frequency drift forces, as well. The phenomenon of drift force is described in the frequency-domain, therefore the time-domain method described in Section 2.2 to compute the potentials can be used if one takes harmonic waves and motions as input signals. In this paper we make use of the results of the codes described in Section 2.1 or 2.2.

The constant and low-frequency drift forces and moments influenced by forward speed can be computed by means of pressure integration along the actual hull. This pressure integration containes the wave frequencies and their sum and difference frequencies. To obtain the low-frequency and constant second-order forces and moments the wave frequencies and their sum frequencies are filtered out. This procedure is easily carried out in the time-domain. First we discuss the method consisting of pressure integration over the actual ship hull. The mean drift force can also be obtained by means of application of conservation of momentum in the fluid domain; this leads to expressions where the far-field evaluation of the first order potentials are used. This method is discussed secondly.

3.1. Constant and low-frequency drift forces by means of local expansions

First we discuss the second-order slowly varying drift forces. The forces acting on the hull can be obtained by integration of the pressure along the exact wetted hull surface $\tilde{S}$. The pressure on the surface is given by Bernoulli's equation

$$p(\boldsymbol{x}, t) = -\rho\left(\frac{\partial \Phi}{\partial t} + \tfrac{1}{2}\nabla\Phi \cdot \nabla\Phi + gz - \tfrac{1}{2}U^2\right) \quad \text{on } \tilde{S}.$$

The force acting on the hull is obtained by integration of the pressure over the exact surface

$$\boldsymbol{F} = \int_{\tilde{S}} p\boldsymbol{n}\,\mathrm{d}S. \tag{12}$$

This may look an easy operation, but it is not, because the linearised potentials are computed at the mean hull surface and the integration goes up to the free water surface, while linearisation with respect to $z = 0$ has taken place. Hence the expression for the force has to be perturbed to integrals over the mean hull surface S and the unperturbed water-line. We assume that the total displacement of a point $\boldsymbol{x}$ of the surface is given in linearised form as

$$\boldsymbol{\alpha} = \boldsymbol{X} + \boldsymbol{\Omega} \times (\boldsymbol{x} - \boldsymbol{x}_g), \tag{13}$$

with $\boldsymbol{X}$ the translational and $\boldsymbol{\Omega}$ the rotational motion of the body relative to its center of gravity $\boldsymbol{x}_g$. At this point several assumptions are made, but some of them are more or less questionable. The pressure is expanded in a Taylor series around the average surface S. This can be done because the pressure is a differentiable function of $\boldsymbol{x}$.

$$p_{\tilde{S}} = p_S + \boldsymbol{\alpha} \cdot \nabla p_S + \tfrac{1}{2}(\boldsymbol{\alpha} \cdot \nabla)^2 p_S + \mathcal{O}(|\boldsymbol{\alpha}|^3). \tag{14}$$

A step that poses some geometrical limitations, like a vertical ship hull at the free surface, on the applicablity of this approach is the evaluation of the integral

$$\int_{\tilde{S}} f(\boldsymbol{x})\,\mathrm{d}S \approx \int_S f(\boldsymbol{x})\,\mathrm{d}S + \int_{wl}\int_{\alpha_3}^{\zeta} f(\boldsymbol{x})\,\mathrm{d}z\,\mathrm{d}l. \tag{15}$$

We assume the height of the waves and the motions of the hull to be small $\mathcal{O}(\varepsilon)$, hence we expand all quantities with respect to ε as follows

$$\Phi(\boldsymbol{x}, t) = \overline{\Phi}(\boldsymbol{x}) + \varepsilon\psi^{(1)}(\boldsymbol{x}, t) + \varepsilon^2\psi^{(2)}(\boldsymbol{x}, t) + \mathcal{O}(\varepsilon^3),$$

$$\zeta(x, y, t) = \overline{\zeta}(x, y) + \varepsilon\zeta^{(1)}(x, y, t) + \varepsilon^2\zeta^{(2)}(x, y, t) + \mathcal{O}(\varepsilon^3),$$

$$p_S = \overline{p}(\boldsymbol{x}) + \varepsilon p^{(1)}(\boldsymbol{x}, t) + \varepsilon^2 p^{(2)}(\boldsymbol{x}, t) + \mathcal{O}(\varepsilon^3),$$

$$\boldsymbol{X} = \varepsilon\boldsymbol{X}^{(1)} + \varepsilon^2\boldsymbol{X}^{(2)} + \mathcal{O}(\varepsilon^3),$$

$$\boldsymbol{\Omega} = \varepsilon\boldsymbol{\Omega}^{(1)} + \varepsilon^2\boldsymbol{\Omega}^{(2)} + \mathcal{O}(\varepsilon^3),$$

$$\boldsymbol{x} - \boldsymbol{x}_g = \overline{\boldsymbol{x}} - \boldsymbol{x}_g + \varepsilon\boldsymbol{\Omega}^{(1)} \times (\overline{\boldsymbol{x}} - \boldsymbol{x}_g) + \varepsilon^2\boldsymbol{\Omega}^{(2)} \times (\overline{\boldsymbol{x}} - \boldsymbol{x}_g) + \mathcal{O}(\varepsilon^3),$$

$$\boldsymbol{n} = \overline{\boldsymbol{n}} + \varepsilon\boldsymbol{\Omega}^{(1)} \times \overline{\boldsymbol{n}} + \varepsilon^2\boldsymbol{\Omega}^{(2)} \times \overline{\boldsymbol{n}} + \mathcal{O}(\varepsilon^3),$$

where the second-order terms like $\psi^{(2)}(\boldsymbol{x}, t)$ also contain a stationary part due to the quadratic terms of wave components with itself. The first terms in the perturbation series are time independent, so

$$\overline{\zeta} = \tfrac{1}{2}(\nabla\overline{\Phi} \cdot \nabla\overline{\Phi} - U^2)$$

is the stationary wave height. We consider small steady velocities which are considered to be small enough to linearise the free surface as we described. Substituting in the Taylor series for the pressure at the actual hull surface and collecting equal powers of ε, we get

$$
\begin{aligned}
\overline{p}_{\tilde{S}} &= \overline{p}, \\
p_{\tilde{S}}^{(1)} &= p^{(1)} + \{\boldsymbol{X}^{(1)} + \boldsymbol{\Omega}^{(1)} \times (\overline{\boldsymbol{x}} - \boldsymbol{x}_g)\} \cdot \nabla \overline{p}, \\
p_{\tilde{S}}^{(2)} &= p^{(2)} + \{\boldsymbol{X}^{(2)} + \boldsymbol{\Omega}^{(2)} \times (\overline{\boldsymbol{x}} - \boldsymbol{x}_g) + \boldsymbol{\Omega}^{(1)} \times [\boldsymbol{\Omega}^{(1)} \times (\overline{\boldsymbol{x}} - \boldsymbol{x}_g)]\} \cdot \nabla \overline{p} \\
&\quad + \{\boldsymbol{X}^{(1)} + \boldsymbol{\Omega}^{(1)} \times (\overline{\boldsymbol{x}} - \boldsymbol{x}_g)\} \cdot \nabla p^{(1)} \\
&\quad + \tfrac{1}{2}\{[\boldsymbol{X}^{(1)} + \boldsymbol{\Omega}^{(1)} \times (\overline{\boldsymbol{x}} - \boldsymbol{x}_g)] \cdot \nabla\}^2 \overline{p}.
\end{aligned}
$$

From Bernoulli and the perturbation series for the potentials we get for the components of the pressure on the mean wetted surface

$$
\begin{aligned}
&\overline{p} = -\rho(gz_0 + \tfrac{1}{2}(\nabla\overline{\Phi} \cdot \nabla\overline{\Phi} - U^2)), \\
&p^{(1)} = -\rho\left(\nabla\overline{\Phi} \cdot \nabla\psi^{(1)} + \frac{\partial \psi^{(1)}}{\partial t}\right), \\
&p^{(2)} = -\rho\left(\frac{\partial \psi^{(2)}}{\partial t} + \tfrac{1}{2}\nabla\psi^{(1)} \cdot \nabla\psi^{(1)} + \nabla\psi^{(2)} \cdot \nabla\overline{\Phi}\right).
\end{aligned}
$$

We carry out the pressure integration along the hull and apply high-frequency filtering in the case of the multi-frequency case and averaging in the case of one harmonic wave. We obtain for the second-order low-frequency or constant drift force

$$
\begin{aligned}
\boldsymbol{F}^{(2)} &= \tfrac{1}{2}\rho g \int_{WL} (\zeta^{(1)} - \alpha_3^{(1)})^2 \overline{\boldsymbol{n}}\, \mathrm{d}l + \boldsymbol{\Omega}^{(1)} \times M \frac{\mathrm{d}^2 \boldsymbol{X}^{(1)}}{\mathrm{d}t^2} \\
&\quad -\rho \int_S \left\{ (\boldsymbol{\alpha}^{(1)} \cdot \nabla) \left(\frac{\partial \psi^{(1)}}{\partial t} + \nabla\psi^{(1)} \cdot \nabla\overline{\Phi} \right) + \tfrac{1}{2}\nabla\psi^{(1)} \cdot \nabla\psi^{(1)} \right. \\
&\qquad \left. + \frac{\partial \psi^{(2)}}{\partial t} + \nabla\psi^{(2)} \cdot \nabla\overline{\Phi} \right\} \overline{\boldsymbol{n}}\, \mathrm{d}S. \qquad (16)
\end{aligned}
$$

For the second-order moment we obtain a similar expression

$$
\begin{aligned}
\boldsymbol{M}^{(2)} &= \tfrac{1}{2}\rho g \int_{WL} (\zeta^{(1)} - \alpha_3^{(1)})^2 (\overline{\boldsymbol{x}} - \boldsymbol{x}_g) \times \overline{\boldsymbol{n}}\, \mathrm{d}l + \boldsymbol{\Omega}^{(1)} \times M \frac{\mathrm{d}^2 \boldsymbol{\Omega}^{(1)}}{\mathrm{d}t^2} \\
&\quad -\rho \int_S \left\{ (\boldsymbol{\alpha}^{(1)} \cdot \nabla) \left(\frac{\partial \psi^{(1)}}{\partial t} + \nabla\psi^{(1)} \cdot \nabla\overline{\Phi} \right) + \tfrac{1}{2}\nabla\psi^{(1)} \cdot \nabla\psi^{(1)} \right. \\
&\qquad \left. + \frac{\partial \psi^{(2)}}{\partial t} + \nabla\psi^{(2)} \cdot \nabla\overline{\Phi} \right\} (\overline{\boldsymbol{x}} - \boldsymbol{x}_g) \times \overline{\boldsymbol{n}}\, \mathrm{d}S. \qquad (17)
\end{aligned}
$$

The influence of the second order potentials in both the second-order forces and moments is considerable if one considers finite depth. Their influence on the second-order constant drift forces is negligible as follows from the analysis based on the far-field approximations, while in the computations of the constant second-order moments they may not neglected as is described by Grue [15]. Many available computer codes take care of these effects in an approximate way, see, for instance, the work of Pinkster [11].

3.2. Constant drift forces by means of far-field expansions

Here we are mainly interested in the constant component of the drift force. In this section we apply a method that leads to results that are more accurate numerically. The direct pressure integration needs velocities obtained by differentiation of the potentials, depending on the numerical method to obtain the source stengths. This may lead to results that are not accurate enough for our purpose. Our choice is not to go to higher-order panel methods in the solver, but to use the results for the potentials and to avoid differentiation by applying the conservation of impulse for the fluid domain. This method is the one that in the past led to the first results of the drift forces. Maruo [6] and later Newman [7] have derived an expression for the wave-drift forces and moments in still water. In this paper we restrict ourselves to the determination of the mean drift forces. For a discussion of the drift moment in current we refer to Grue and Palm [34].

The components of the horizontal mean drift forces, $\overline{F}_x$ and $\overline{F}_y$,

$$\overline{F}_x = -\overline{\iint_{S_\infty} [p\cos\theta + \rho V_R(V_R\cos\theta - V_\theta\sin\theta)]R\,\mathrm{d}\theta\,\mathrm{d}z}, \tag{18}$$

$$\overline{F}_y = -\overline{\iint_{S_\infty} [p\sin\theta + \rho V_R(V_R\sin\theta + V_\theta\cos\theta)]R\,\mathrm{d}\theta\,\mathrm{d}z}, \tag{19}$$

where p is the first-order hydrodynamic pressure, $\mathbf{V}$ is the fluid velocity with radial and tangential components V_R ,V_θ and S_∞ is a large cylindrical control surface with radius R in the ship-fixed coordinate system. Faltinsen and Michelsen [8] derive from these formulas expressions in terms of the source densities of the first-order potentials in the case of zero speed at finite depth. We follow a similar approach from

$$\sigma = \sigma^{(7)} + \sum_{j=1}^{6} \sigma^{(j)}\overline{\alpha}_j,$$

where $\alpha_j = \overline{\alpha}_j\,\mathrm{e}^{-i\omega t}$, $j = 1(1)6$ are the six modes of motion and the superscript 7 refers to the diffracted component of the source strength. However, in our case, the velocity potential has the form

$$\begin{aligned}\Phi(\boldsymbol{x},t) &= Ux + \overline{\phi}(\boldsymbol{x};U) + \phi(\boldsymbol{x};U)\,\mathrm{e}^{-i\omega t}\\ &= Ux + \overline{\phi}(\boldsymbol{x};U) + \left\{\phi^{\mathrm{inc}}(\boldsymbol{x};U) + \phi^{(7)}(\boldsymbol{x};U) + \sum_{j=1}^{6}\phi^{(j)}(\boldsymbol{x};U)\overline{\alpha}_j\right\}\mathrm{e}^{-i\omega t},\end{aligned} \tag{20}$$

where the potentials $\phi^{(j)}(\boldsymbol{x};U)$, $j = 1, 7$ have the form (4) and are the potentials due to the motions and the diffraction. We assume that the potentials and the source strengths are expressed in terms of perturbation series (6, 7), and that the first two terms are known at this stage.

In the far field $R \gg 1$ we neglect the influence of the stationary potential $\overline{\phi}(\boldsymbol{x}; U)$ in (20); hence we approximate (20) by

$$\Phi(\boldsymbol{x}, t) = Ux + \frac{g\zeta_a}{\omega_0} \exp\{k_1(\beta)z + i[k_1(\beta)(x\cos\beta + y\sin\beta) - i\omega t]\}$$
$$+F(\theta; U)\,\mathrm{e}^{iS(\theta;U)}\sqrt{\frac{1}{R}}\exp\{k_1(\theta)z + i[k_1(\theta)R - i\omega t]\}. \tag{21}$$

Here ζ_a is the amplitude of the incoming wave in the direction β and the wave number $k_1(\beta) = k_0$. However, in the local coordinates we write

$$k_1(\beta) = \frac{g + 2\omega U\cos\beta - g\sqrt{1 + 4\tau\cos\beta}}{2U^2\cos^2\beta} \approx \tilde{k}(1 - 2\tau\cos\beta) + O(\tau^2) = k_0 + O(\tau^2),$$

where we use the notation $\tilde{k} = \omega^2/g$. Notice that this wave number is defined in the ship-fixed coordinate system and is different from $k_0 = \omega_0^2/g$, as defined before in the earth-fixed coordinate system. The function $F(\theta)\,\mathrm{e}^{iS(\theta)}$ results from the asymptotic expansion of the far field potentials in (20) with

$$4\pi\phi^{(j)}(\boldsymbol{x}; U) = \iint_S \sigma^{(j)}(\boldsymbol{\xi})\Psi(\boldsymbol{x}, \boldsymbol{\xi})\,\mathrm{d}S_\xi$$
$$-\frac{2i\omega}{g}\iint_{FS} \nabla\overline{\phi}\cdot\nabla\phi^{(j)}\Psi(\boldsymbol{x}, \boldsymbol{\xi})\,\mathrm{d}S_\xi, \tag{22}$$

where $\Psi(\boldsymbol{x}, \boldsymbol{\xi})$ is the asymptotic expansion of the Green's function in the far field

$$\Psi(\boldsymbol{x}, \boldsymbol{\xi}) \approx 2\pi i\sqrt{\frac{2}{\pi R}}\;\frac{\exp\{i[k_1(\theta)R - \frac{\pi}{4}]\}}{[1 + \frac{2U}{g}(\omega - k_1(\theta)U\cos\theta)\cos\theta]}$$
$$\times\sqrt{k_1(\theta)}\exp\{k_1(\theta)(z + \zeta) - ik_1(\theta)(\xi\cos\theta + \eta\sin\theta)\}, \tag{23}$$

with the local $k_1(\theta)$ wave number defined as

$$k_1(\theta) = \frac{g + 2\omega U\cos\theta - g\sqrt{1 + 4\tau\cos\theta}}{2U^2\cos^2\theta}. \tag{24}$$

Due to the fact that the function $\nabla\overline{\phi}(\boldsymbol{\xi})$ decays rapidly as $|\boldsymbol{\xi}| \to \infty$, it can be shown that the last term in (22) is approximated correctly with $\nabla\phi^{(j)}$ replaced by $\nabla\phi_0^{(j)}$. This leads to

$$F(\theta; U)\,\mathrm{e}^{iS(\theta;U)}$$
$$= \sqrt{\frac{k_1(\theta)}{2\pi}}\left\{\frac{\mathrm{e}^{\pi i/4}}{1 + \frac{2U}{g}(\omega - k_1(\theta)U\cos\theta)\cos\theta}\right\}$$

$$\times \left\{ \iint_S (\sigma_0(\boldsymbol{\xi}) + \tau\sigma_1(\boldsymbol{\xi}) + \cdots) \exp\{k_1(\theta)\zeta - ik_1(\theta)(\xi\cos\theta + \eta\sin\theta)\}\, \mathrm{d}S_\xi \right.$$

$$\left. - 2i\tau \iint_{FS} \frac{\nabla\overline{\phi}}{U} \cdot \nabla\phi_0^{(T)} \exp\{-ik_1(\theta)(\xi\cos\theta + \eta\sin\theta)\}\, \mathrm{d}S_\xi \right\}, \tag{25}$$

where

$$\phi_0^{(T)} = \phi_0^{(7)} + \sum_{j=1}^{6} \phi_0^{(j)} \overline{\alpha}_j .$$

It is obvious that (25) can be written in the form

$$F(\theta; U)\, \mathrm{e}^{iS(\theta;U)} = (1 - 2\tau\cos\theta) F_0(\theta)\, \mathrm{e}^{iS_0(\theta)} + \tau F_1(\theta)\, \mathrm{e}^{iS_1(\theta)} + O(\tau^2). \tag{26}$$

The functions $F_i(\theta)$ and $S_i(\theta)$ contain the local wave number $k_1(\theta)$. The upper boundary of integration in (18) and (19) is the free surface

$$\zeta = \frac{1}{g} \Re[(i\omega\phi(x, y, 0) - U\phi_x(x, y, 0))\, \mathrm{e}^{-i\omega t}].$$

It follows from the pressure term that we can write

$$\overline{\int_{-\infty}^{\zeta} p dz} = \overline{\tfrac{1}{2}\rho g \zeta^2} - \tfrac{1}{2}\rho \overline{\int_{-\infty}^{0} (|\, \mathbf{V}\,|^2 - U^2)\, \mathrm{d}z}.$$

We find the following expression for $\overline{F}_x$

$$\begin{aligned} \overline{F}_x \;=\; & -\tfrac{1}{4}\rho \int_0^{2\pi} \tilde{k}\phi\phi^* R\cos\theta\, \mathrm{d}\theta + \tfrac{1}{2}\rho\tau \int_0^{2\pi} \Im(\phi\phi_x^*\cos\theta + \phi\phi_y^*\sin\theta) R\, \mathrm{d}\theta \\ & + \tfrac{1}{4}\rho \int_0^{2\pi} \int_{-\infty}^{0} \left[\left(\frac{1}{R^2}\phi_\theta\phi_\theta^* - \phi_R\phi_R^* + \phi_z\phi_z^* \right) R\cos\theta \right. \\ & \left. + (\phi_R\phi_\theta^* + \phi_\theta\phi_R^*)\sin\theta \right] \mathrm{d}\theta\, \mathrm{d}z \end{aligned} \tag{27}$$

and for $\overline{F}_y$:

$$\begin{aligned} \overline{F}_y \;=\; & -\tfrac{1}{4}\rho \int_0^{2\pi} \tilde{k}\phi\phi^* R\sin\theta\, \mathrm{d}\theta - \tfrac{1}{2}\rho\tau \int_0^{2\pi} \Im(\phi\phi_x^*\sin\theta - \phi\phi_y^*\cos\theta) R\, \mathrm{d}\theta \\ & + \tfrac{1}{4}\rho \int_0^{2\pi} \int_{-\infty}^{0} \left[\left(\frac{1}{R^2}\phi_\theta\phi_\theta^* - \phi_R\phi_R^* + \phi_z\phi_z^* \right) R\sin\theta \right. \\ & \left. - (\phi_R\phi_\theta^* + \phi_\theta\phi_R^*)\cos\theta \right] \mathrm{d}\theta\, \mathrm{d}z. \end{aligned}$$

In these expressions the asterisks denote the complex conjugate. The integration with respect to z needs some extra attention due to the fact that the exponential behaviour of the incident wave and the radiated or diffracted wave is different, due to the dependence of the wave number on β and θ, respectively. The function ϕ follows from (21)

$$\phi = \frac{g\zeta_a}{\omega_0} \exp\{k_1(\beta)z + ik_1(\beta)(x\cos\beta + y\sin\beta)\}$$

$$+F(\theta; U)\,\mathrm{e}^{iS(\theta;U)}\sqrt{\frac{1}{R}}\exp\{k_1(\theta)z + ik_1(\theta)R\}. \tag{28}$$

A closer look at (28) and (27) shows that the contributions to $\overline{F}_x$ consist of two parts. The first one, $F_x^{(1)}$, originates from those parts of the cross products that behave like $R^{-1/2}$, while the second one, $F_x^{(2)}$, originates from those square terms in (27) that behave like R^{-1}. We formally write

$$\overline{F}_x = F_x^{(1)} + F_x^{(2)}. \tag{29}$$

After some lengthy manipulations and asymptotic expansion for large values of R we obtain for the mean surge force $F_x^{(1)}$ and the mean sway force $F_y^{(1)}$

$$F_x^{(1)} \approx \mathcal{A}\sqrt{\frac{2\pi}{\tilde{k}}}F(\beta^*; U)\cos(S(\beta^*; U) + \tfrac{1}{4}\pi)\cos\beta + O(\tau^2) \tag{30}$$

and

$$F_y^{(1)} \approx \mathcal{A}\sqrt{\frac{2\pi}{\tilde{k}}}F(\beta^*; U)\cos(S(\beta^*; U) + \tfrac{1}{4}\pi)\sin\beta + O(\tau^2), \tag{31}$$

where

$$\mathcal{A} = -\frac{\rho\omega^2}{2\omega_0}\zeta_a \quad \text{and} \quad \beta^* = \beta - 2\tau\sin\beta.$$

The second part of the wave-drift force may be analysed in the same way. We obtain

$$F_x^{(2)} \approx -\tfrac{1}{4}\rho\tilde{k}\int_0^{2\pi} F^2(\theta; U)\{\cos\theta - 2\tau\sin^2\theta\}\,\mathrm{d}\theta + O(\tau^2) \tag{32}$$

and

$$F_y^{(2)} \approx -\tfrac{1}{4}\rho\tilde{k}\int_0^{2\pi} F^2(\theta; U)\{\sin\theta(1 + 2\tau\cos\theta)\}\,\mathrm{d}\theta + O(\tau^2).$$

For the zero-speed case this result is in accordance with Maruo [6] and for the general situation with Nossen *et al*. [35] if we change some signs due to the fact that our ship is sailing to the left. The formulation of the second-order moment acting on a vessel with constant forward speed and waves can be derived in a similar way [36]. Grue [17] also studied the situation where the vessel rotates slowly. This is of importance in case the vessel, moored to a single point, rotates slowly with large yawing angles. As noticed in the experimental work of Wichers [12] for the description of the low-frequency yaw motions potential effect are of a minor importance. The flow is largely dominated by viscous effects, especially in the case flow separation occurs. To determine these motions one has to rely on experiments both for the excitation and the coefficients in the equations of motion.

4. Wave-drift damping

We now consider the situation that a ship is moored to a soft spring system in waves. We consider the motion, due to head seas, in the longitudinal direction, which is the x-direction. Due to the low-frequency drift force the ship is excited in the eigen-frequency of the moored system. The damping coefficient for this low frequency oscillatory drift motion results from the speed dependency of the drift force. One must realise that this hypothesis is to be checked. In fact the damping equals to the out-of-phase, with respect to the motion, part of the slowly oscillating drift force. In a paper by Newman [37] the formal solution of this problem is described. Due to the fact that the forward velocity is low, we assume that an approximation of this damping can be obtained by the following approach

$$b_{xx} = -\frac{\partial \overline{F}_x}{\partial U}|_{U=0}, \tag{33}$$

where $\overline{F}_x$ is computed in regular waves with fixed wave frequency. We can use the results of Section 3.1 or the results of Section 3.2, where the drift forces are described at constant forward speed. Aranha [38] has presented an approximation for the wave drift damping of the following form

$$b_{xx} = k_0 \frac{\partial \overline{F}_x(\omega_0)}{\partial \omega_0} + \frac{4\omega_0}{g}\overline{F}_x(\omega_0). \tag{34}$$

This expression is sometimes approximated at high frequencies by

$$b_{xx} \approx \frac{4\omega_0}{g}\overline{F}_x(\omega_0). \tag{35}$$

However, Equation (34) is derived for the two-dimensional case and in three-dimensions it is not fully justified. In some cases it leads to a good numerical approximation. A drawback is that the dependency of the drift forces on the first-order-motion amplitudes and phases is not properly taken into account. This may lead to a large deviation especially if the drift force is rather peaked due to motion effects. We combine (30) with (32) and make use of conservation of energy, as suggested by Maruo [6], and obtain

$$\begin{aligned}
&\overline{F}_x(\omega; U) \\
&\quad \approx -\tfrac{1}{4}\rho\tilde{k}\int_0^{2\pi} F^2(\theta; U)(\cos\theta - \cos\beta - 2\tau\sin^2\theta)\,\mathrm{d}\theta + O(\tau^2) \\
&\quad \approx \overline{F}_x(\omega_0)(1+2\tau) + \tfrac{1}{2}\rho k_0 \tau \int_0^{2\pi} \left\{ \sin^2\theta\, F^2(\theta) - \frac{g}{\omega_0}F(\theta; U) \right. \\
&\qquad\qquad\qquad \left. \times \frac{\partial F(\theta; U)}{\partial U}(\cos\theta - \cos\beta) \right\} \mathrm{d}\theta.
\end{aligned} \tag{36}$$

This formulation can be rewritten with (26). In this case we have

$$\begin{aligned}\overline{F}_x(\omega; U) \approx & -\tfrac{1}{4}\rho\tilde{k}\int_0^{2\pi} F_0^2(\theta)(\cos\theta - \cos\beta)\,\mathrm{d}\theta \\ & +\tfrac{1}{4}\rho\tilde{k}\tau\int_0^{2\pi}\{4\cos\theta(\cos\theta-\cos\beta) - 2\sin\theta^2\}F_0^2(\theta)\,\mathrm{d}\theta \\ & -\tfrac{1}{2}\rho\tilde{k}\tau\int_0^{2\pi} F_0(\theta)F_1(\theta)\cos(S_0(\theta)-S_1(\theta)) \\ & \times(\cos\theta-\cos\beta)\,\mathrm{d}\theta + O(\tau^2). \end{aligned} \tag{37}$$

If we use Equation (36), the damping in the x-direction can be written as follows

$$\begin{aligned} b_{xx} = & \frac{2\omega_0}{g}\left\{\overline{F}_x(\omega_0) + \frac{\rho k_0}{4}\int_0^{2\pi}\left\{\sin^2\theta F_0^2(\theta) - \frac{g}{\omega_0}F_0(\theta)\right.\right. \\ & \left.\left.\times\frac{\partial F(\theta;0)}{\partial U}(\cos\theta-\cos\beta)\right\}\mathrm{d}\theta\right\}. \end{aligned} \tag{38}$$

In the same way we find an expression for b_{xy}

$$\begin{aligned} b_{xy} = & \frac{2\omega_0}{g}\left\{\overline{F}_y(\omega_0) - \frac{\rho k_0}{4}\int_0^{2\pi}\left\{\sin\theta\cos\theta F_0^2(\theta) + \frac{g}{\omega_0}F_0(\theta)\right.\right. \\ & \left.\left.\times\frac{\partial F(\theta;0)}{\partial U}(\sin\theta-\sin\beta)\right\}\mathrm{d}\theta\right\}. \end{aligned} \tag{39}$$

With the time-domain computer program we carried out the computations for the drift forces and wave-drift damping in an independent way. The results are shown in Figure 2; the drift forces for the sphere at zero and small positive and negative forward speed are shown. In Figure 3 the wave-drift damping obtained by the approximate formula (34) and the numerical results are shown. Also, the graph of $(4\omega_0/g)\overline{F}_x(\omega_0)$ is shown. The high-frequency limit of the computed wave-drift damping coincides with the asymptotic value as obtained from the derivative at $\tau = 0$ of the drift force obtained by an asymptotic ray method [39]; this is a good check of the numerical results. The results for a VLCC-tanker are for the wave-drift force shown in Figure 4 and for the corresponding wave-drift damping in Figure 5. It is clear that, due to resonance in the motion that leads to a large negative gradient in the wave-drift force locally, the approximate formula is in disagreement with the results computed by the direct method. The results for wave-drift forces on a LNG-gas carrier are shown in Figure 6. In this case also the results for rather large values of the forward speed are shown. For this reason upwind differencing has been used in the time-domain code. The results for the wave-drift damping in Figure 7 show good agreement with the experiments carried out at MARIN by Wichers [12]. In Figure 8 the drift force for a semisubmersible is shown. The computations and experiments were carried out at MARIN by Huijsmans [23]. The computer program is a frequency-domain program on the basis of a source distribution with the Green's function at zero speed as source term. In general, one may say that the computational results fit well with the experimental results. One must keep in mind that accurate measurements are hard to obtain, as can be seen in the scatter of the results by repeated experiments with different wave heights.

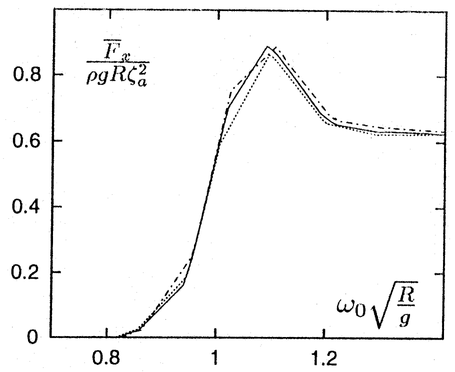

Figure 2. Wave-drift force for the sphere, — $Fn = 0$, · — · — · — $Fn = 0{\cdot}005$, · · · · · $Fn = -0{\cdot}005$.

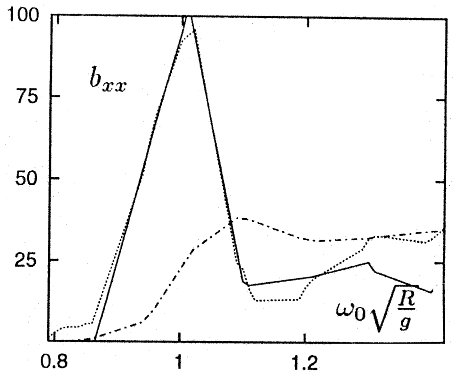

Figure 3. Wave-drift damping for the sphere with $R = 10$, · · · · · · (34), · — · — · — (35), — direct computation.

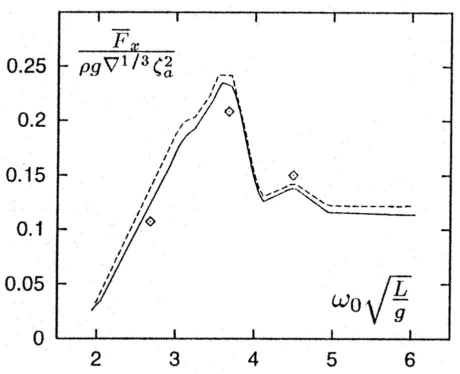

Figure 4. Wave-drift force for a tanker with, — $Fn = 0$, — $Fn = 0{\cdot}004$, ◊ measurements by Wichers [12].

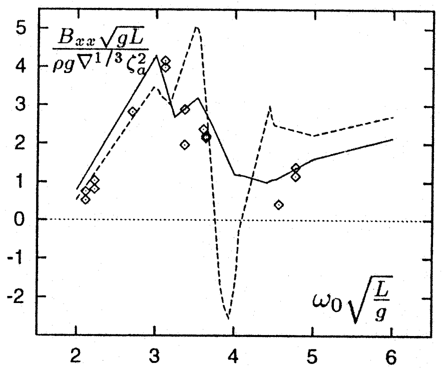

Figure 5. Wave-drift damping for a tanker, — direct computations, – – – (34), ◊ measurements by Wichers [12].

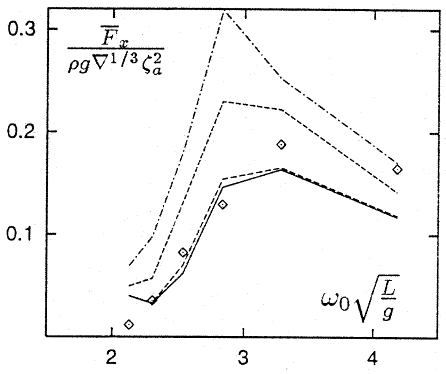

Figure 6. Wave-drift force for a LNG-carrier with, — $Fn = 0$, – – – $Fn = 0{\cdot}003$, – – – $Fn = 0{\cdot}025$, · – · – · – $Fn = 0{\cdot}05$, ◊ measurements by Wichers [12].

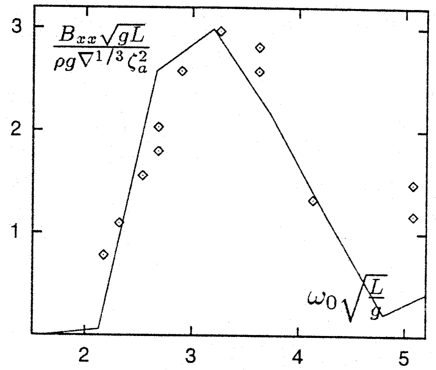

Figure 7. Wave-drift damping for a LNG-carrier, — direct computations, ◊ measurements by Wichers [12].

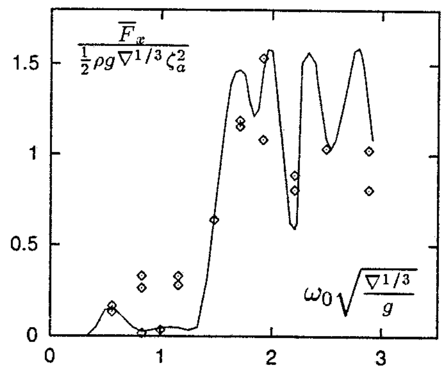

Figure 8. Wave-drift force for a semi submersible for $Fn = 0$ with measuremnts ◊ by Huijsmans [23].

5. Conclusions

For the description of the low-frequency motions of large moored structures several methods exist. In this paper we have discussed the excitation forces due to waves. These forces can be derived by means of potential theory. The reaction forces, however, consist of a potential and a viscous part. The latter is not addressed in this paper and we have restricted ourselves to the potential part. This part becomes significant especially in survival seastates, due to the fact that it is proportional to the square of the wave height. To compute these second-order effects one has to compute the first-order forces and motions first. This can be done either in the frequency- or the time domain. The wave-drift forces and motions are computed in the frequency domain. A few computer codes that can compute these low-frequency motions are available at this moment.

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